The Hamiltonian of a charged particle in a magnetic field is

$$H = \frac{1}{2m}\left(P - \frac{e}{c}A\right)^2 .$$

Given the magnetic field $\bar{B} = \nabla \times A$ is a constant, say $\bar{B} = B_0 \hat{k}$ along z-axis.

$$\Rightarrow A = -\frac{1}{2}(\bar{r} \times \bar{B}) = -\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 0 & 0 & B_0 \end{vmatrix}$$

$$= -\frac{1}{2}B_0(y\hat{i} - x\hat{j})$$

$$\Rightarrow A_x = \frac{-B_0}{2}y, \quad A_y = \frac{B_0 x}{2}, \quad A_z = 0.$$

$\therefore$ Hamiltonian,

$$H = \frac{1}{2m}\left[\left(P_x - \frac{e}{c}A_x\right)^2 + \left(P_y - \frac{e}{c}A_y\right)^2 + \left(P_z - \frac{e}{c}A_z\right)^2\right]$$

$$= \frac{1}{2m}\left[\left(P_x + \frac{e}{c}\frac{B_0 y}{2}\right)^2 + \left(P_y - \frac{eB_0}{2c}x\right)^2 + P_z^2\right].$$

a) The partition function is,

$$Z = \frac{1}{h^{3N}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d^3p_1 \cdots d^3p_N \, d^3q_1 \cdots d^3q_N \; e^{-\frac{\beta}{2m} \sum_{i=1}^{N} \left[\left(p_x^i + \frac{e}{c}\frac{B_0 y^i}{2}\right)^2 + \left(p_y^i - \frac{eB_0}{2c} x^i\right)^2 + p_z^{i\,2} \right]}$$

where i indicates i^{th} particle's momentum / position.

$$\Rightarrow Z = \frac{1}{h^{3N}} \left[\int_{-\infty}^{\infty} dx\,dy\,dz\,dp_x\,dp_y\,dp_z \; e^{-\frac{\beta}{2m}\left(p_x + \frac{eB_0 y}{2c}\right)^2 - \frac{\beta}{2m}\left(p_y - \frac{eB_0}{2c}x\right)^2 - \frac{\beta p_z^2}{2m}} \right]^N$$

Let do a change of integration variable for p_x, p_y

$\Rightarrow q_x = p_x + \frac{eB_0}{2c} y, \quad q_y = p_y - \frac{eB_0}{2c} x, \quad q_z = p_z$

with integration limits $-\infty$ to ∞. and Jacobian is one.

$$\Rightarrow Z = \frac{1}{h^{3N}} \left[\int_{-\infty}^{\infty} dx\,dy\,dz\,dq_x\,dq_y\,dq_z \; e^{-\frac{\beta}{2m}\left(q_x^2 + q_y^2 + q_z^2\right)} \right]^N$$

$$= \frac{1}{h^{3N}} \left[\int_{-\infty}^{\infty} dx\,dy\,dz \right]^N \left[\int_{-\infty}^{\infty} dq_x\,dq_y\,dq_z \; e^{-\frac{\beta}{2m}\left(q_x^2+q_y^2+q_z^2\right)} \right]^N$$

$$= \frac{V^N}{h^{3N}} \left[\left(\frac{\sqrt{\pi}}{\sqrt{\beta/2m}} \right)^3 \right]^N, \quad \text{where } V \text{ is the volume of the cube.}$$

$$\Rightarrow Z = \left[\frac{V}{h^3} \left(\frac{2\pi m}{\beta} \right)^{3/2} \right]^N, \quad \text{where } \beta \text{ is the inverse temperature, } \beta = 1/k_B T$$

b) The Magnetisation,

$M = \frac{\mu}{V}$, where total magnetisation

$$\mu = -\left(\frac{\partial F}{\partial B}\right)_T = -\frac{\partial}{\partial B}\left(k_B T \log Z\right) = \frac{\partial}{\partial B}\left(k_B T \log Z\right) = 0$$

$\Rightarrow$ magnetisation is zero for classical charged particle. It is a quantum phenomenon.