Answer :-

Given that - single particle of mass m in large box volume V.

a) when the box is in equilibrium with temperature T. The partition function as follows

$$Z = \sum_i e^{-\beta E_i} \qquad \text{where } \beta = \frac{1}{K_B T}$$

$$Z = e^{-\left(\frac{-V_0}{K_B T}\right)} = \exp\left(\frac{V_0}{K_B T}\right)$$

b) when the box become unbound. then the partition function as follows

$$Z = \int_0^\infty d\varepsilon \, f(\varepsilon) \, e^{-\beta\varepsilon}$$

$$f(\varepsilon) = \frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}$$

$$Z = \frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty d\varepsilon \sqrt{\varepsilon}\; e^{-\beta\varepsilon}$$

$$Z = \frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{\sqrt{\pi}}{2\beta^{3/2}}$$

$$= V\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}$$

Average energy-

$$U = -\frac{\partial \ln Z}{\partial\beta} = \frac{\partial}{\partial\beta} V \ln\left(\frac{m}{2\pi\hbar^2\beta}\right)^{3/2}$$

$$U = \frac{3}{2}\left[\frac{\partial}{\partial\beta}\ln\beta + \frac{\partial}{\partial\beta}\left(-\ln\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}\right)\right]$$

$$U = \frac{3}{2}\beta^{-1}$$

$$U = 3/2\, k_BT$$

$$U = 3/2\, k_BT$$

$\therefore$ Hence proved particle become unbound