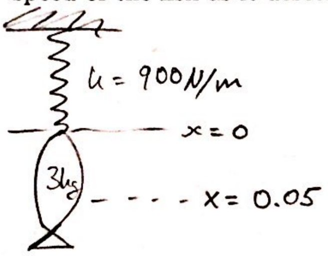
Physics 1A: Physics for Scientists and Engineers: Mechanics **Winter 2015**

Discussion 6: Week 7

Exercise 1 A 3.00 kg fish is attached to the lower end of a vertical spring that has negligible mass and force constant 900 N/m. The spring initially is neither stretched nor compressed. The fish is released from rest. (a) What is its speed after it has descended 0.0500 m from its initial position? (b) What is the maximum speed of the fish as it descends?



a) $E_{tot} = \text{Const.} \Rightarrow \Delta E_{pot} = \Delta E_{spring} + \frac{1}{2}mv^2$

$= \frac{1}{2}k(\Delta x)^2 + \frac{1}{2}mv^2 = +mg\Delta x$

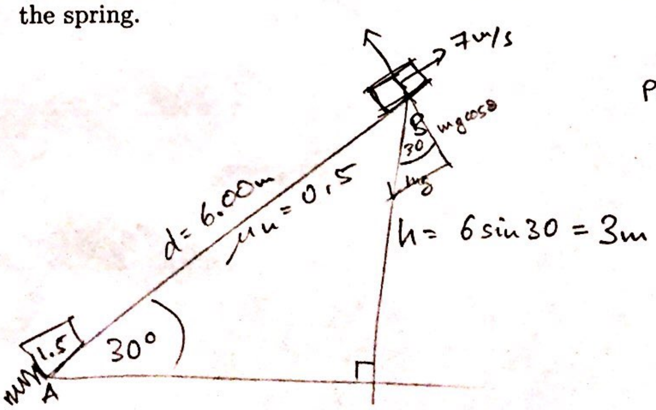
$v = \sqrt{\frac{2}{m}\left(mg\Delta x - \frac{1}{2}k(\Delta x)^2\right)}$

$= \sqrt{2g\Delta x - \frac{k}{m}(\Delta x)^2} = 0.48 \, m/s$

b) $\frac{\partial v}{\partial x} = 0, \quad \frac{\partial^2 v}{\partial x^2} < 0$

$\frac{\partial v}{\partial x} = \frac{1}{2v}\left(2g - \frac{2k\Delta x}{m}\right) = 0 \Rightarrow \Delta x_{...} = \frac{mg}{k}$ @ v_{max}. $\Rightarrow v_{max} = \sqrt{2g\frac{mg}{k} - \frac{mg^2}{k}} = g\sqrt{\frac{m}{k}} = 0.566$

Exercise 2 A wooden block with mass 1.50 kg is placed against a compressed spring at the bottom of an incline of slope 30.0° (point A). When the spring is released, it projects the block up the incline. At point B, a distance of 6.00 m up the incline from A, the block is moving up the incline at 7.00 m/s and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.50$. The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring.



$PE_{spring} = \frac{1}{2}kx^2 = ? = \mu_s N \cdot d + mgh$

$N = mg\cos\theta$

$E_{friction} + E_{grav} = \mu_s mg\cos\theta\, d + mgd\sin\theta$

$= \frac{mgd}{2}(\mu_s\sqrt{3} + 1)$

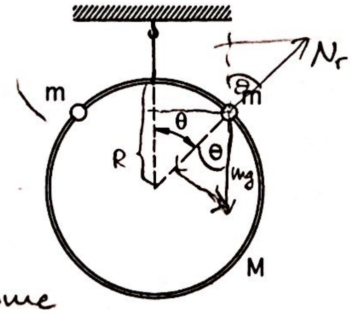
$= 82.4 \, J$

$E_{kin} = \frac{1}{2}mv^2 = 36.8 \, J$

$\Rightarrow PE_{spring} = 119.2 \, J$

$M = \frac{m}{2}$

Exercise 3 A ring of mass $M = 0.1$ kg hangs from a thread, and two beads of mass $m = 0.2$ kg slide on it without friction. The beads are released simultaneously from rest at the top of the ring and slide down opposite sides. The ring is initially motionless, but when the beads pass a critical angle θ_c the ring is observed to start moving upwards. Find the value of θ_c .



When an object undergoes circular motion, its net force is $F_{net,r} = \frac{mv^2}{r}$ in the radial direction.

Here, the ring must provide some radial component of this net force, $\vec{N}$. The radial component of the weight force is $\vec{F}_r = mg\cos\theta\,\hat{r}$.

Together,

$$F_{net,r} = \frac{mv^2}{R} = mg\cos\theta + N \qquad (1)$$

What is $v^2 = ?$

$$\frac{1}{2}mv^2 = mg\Delta h = mg(R - R\cos\theta) \qquad (2)$$

What is $\theta_c = ?$

At some point, the ring must provide a radially inward force on m to keep it in centripetal motion (this point is $\frac{mv^2}{r} = mg\cos\theta_N$, where $\cos\theta_N = \frac{2}{3}$). The masses m will exert an equal and opposite force on the ring (N_r) whose vertical components must cancel the ring's weight (Mg):

$$2N_r\cos\theta_c = Mg \qquad (3)$$

what is $N_r = ?$

$$N_r = \frac{mv^2}{R} - mg\cos\theta_c = \frac{m}{R}\left[2gR(1-\cos\theta_c)\right] - mg\cos\theta_c = 2mg - 3mg\cos\theta_c \qquad (4)$$

$\Rightarrow$ Plug into (3):

$$\Rightarrow -6\cancel{mg}\cos^2\theta_c + 4\cancel{mg}\cos\theta_c - \frac{M\cancel{g}}{m} = 0 \qquad \Rightarrow \cos\theta_c = \frac{-4 \pm \sqrt{16 - 24\frac{M}{m}}}{-12} = \frac{4 \pm 2}{12} = \frac{1}{2}, \frac{1}{6}$$

$$\Rightarrow \theta_c = 60^\circ .$$