STATISTICAL PHYSICS 215A

Final Exam – Spring 2012

Monday 11 June 2012
11:30am – 2:30pm
PAB-2-434

- Please write clearly;
- Present your arguments and calculations clearly;
- All five questions below are independent from one another.

- Print your name on every page used, including this one;
- Make clear which question you are answering on each page;
- No books, notes, computers, or calculators are allowed during the exam;
- Please turn off cell-phones, iPhones, iPods, iPads, Kindles, and other electronic devices.

Grades

Q1.

Q2.

Q3.

Q4.

Q5.

Total /70

QUESTION 1 [14 points]

A furnace contains a gas of N identical molecules in equilibrium at high temperature. Through a small window in the furnace one observes a spectral line of the gas molecules. The width of the observed line is broadened due to the Doppler effect. Derive the intensity $I(\lambda)$ as a function of the observed wavelength λ, the temperature T, the mass m of one molecule, the wavelength λ_0 of the spectral line when the molecule is at rest, the speed of light c, and N.

QUESTION 2 [14 points]

An electron in a magnetic field $\vec{B}$ has energy $\vec{p}^2/2m \pm \kappa B$ according to whether the spin magnetic moment $\vec{\kappa}$ is parallel or anti-parallel to the magnetic field. (Here we set $B = |\vec{B}|$ and $\kappa = |\vec{\kappa}|$.) Calculate the paramagnetic susceptibility χ of a system of free electrons at very low temperatures when the electron gas is completely degenerate.

QUESTION 3 [14 points]

Consider a gas whose equation of state is given by the Van der Waals equation,

$$\left(P + \frac{a}{V^2}\right)(V - b) = NkT$$

with $a, b > 0$ constants.

7 (a) Show that the heat capacity at constant volume C_V depends only on temperature T;
3 (b) Calculate the internal energy E in terms of C_V as a function of T, V, N;
4 (c) Next, suppose that C_V is constant, and that the gas is held in a container of negligible mass which is isolated from its surroundings. Initially, the gas is confined to 1/3 of the total volume of the container by a partition (a vacuum exists in the other 2/3), and is in equilibrium at temperature T_0. Then, a hole is opened in the partition, allowing the gas to irreversibly expand and fill the entire volume V. What is the new temperature of the gas after thermal equilibrium is re-established ?

QUESTION 4 [14 points]

Consider an ideal gas of identical relativistic bosons, whose total number N is conserved. The relation between the energy ε and the momentum $\vec{p}$ of a single boson is given by $\varepsilon = c|\vec{p}|$, where c is the speed of light.

(a) Derive the condition for Bose-Einstein condensation in three space dimensions;
(b) Derive a formula for the critical temperature;
(c) Derive a formula for the fraction of the condensed bosons N_0 to their total number N as a function of temperature, and the other parameters of the problem.
(d) Does Bose-Einstein condensation occur in two space dimensions ? Justify your answer.

QUESTION 5 [14 points]

Atoms in a solid vibrate about their respective equilibrium positions with small oscillations. Debye approximated the normal vibrations with the elastic vibrations of an isotropic body and assumed that the number of vibrational modes $g(\omega)d\omega$ having angular frequency between ω and $\omega + d\omega$ is given by,

$$g(\omega) = 9N\frac{\omega^2}{\omega_D^3}\theta(\omega_D - \omega) \tag{0.1}$$

Here, θ is the Heaviside step function, defined by $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x > 0$, N is the number of atoms, and ω_D is the so-called Debye frequency (which is a constant whose precise value is dependent on the solid).

2 (a) Explain the choice of the normalization factor $9N$ in the function $g(\omega)$ given above.

4 (b) Derive the formulas in terms of $g(\omega)$ for the free energy F and the internal energy E.

4 (c) Calculate the specific heat at constant volume C_V of a solid with the Debye model.

[Do not attempt to evaluate any complicated integrals.]

4 (d) Determine the temperature dependence of C_V at high as well as at low temperatures, and sketch the behavior across all T.