(a) Percolation threshold is a mathematical concept related to percolation theory, which is the formation of long-range connectivity in random systems. below the threshold a giant connected component does not exist;

Conductivity near the percolation threshold is a mixture between a dielectric and a metallic component, the conductivity σ and the dielectric constant ϵ. of this mixture show a critical behaviour if the fraction of the metalic component reaches the percolation threshold.

$$\xi(P) \propto (P_c - P)^{-\nu}$$

$$P(P) \propto (P - P_c)^{\beta}.$$

A one-dimensional chain state of vortex matter. the zero-dimensional analogs of the two-dimensional states that occur at a crystal surface. were observed. at the ends of one-dimensional chain.

(b) In an infinite one-dimensional chain,

The probability n_s that a given site is the left end of a cluster of length precisely s.

P(n arbitrary sites are occupied) $= p^n$.

The probability that a fixed site is to the left end of an s-cluster is therefore $(1-p)^2 p^s$.

Each site has an equal probability of being the left end of an s-cluster. Thus in a system of size L, the total number of s-clusters is, ignoring end effects,

$$L(1-p)^2 p^s.$$

n_s = number of s-clusters per site.

then, $n_s = p^s(1-p)^2$

we also have

$$P(\text{arbitrary site is part of s-cluster}) = n_s s,$$

larger than n_s by a factor of s, since there are s ways an arbitrary site can be situated within an s-cluster

(c) Summing over all possible cluster sizes, we obtain the probability that an arbitrary site is a member of any cluster.

$$\sum_{s=1}^{\infty} n_s s = p \quad (p < p_c).$$

where we write $p < p_c$ to eliminate the possibility of an infinite cluster at p_c.

It is instructive to obtain the previous result from the definition of n_s

$$\sum_s n_s s = \sum_s p^s (1-p)^2 s$$

$$= (1-p)^2 \sum_s p \frac{d}{dp} p^s$$

$$= (1-p)^2 p \frac{d}{dp} \sum_s p^s$$

$$= (1-p)^2 p \frac{d}{dp}\left(\frac{p}{1-p}\right)$$

$$= p$$

(d) Another quantity of interest is

$$\omega_s = P(\text{arbitrary occupied site belongs to an } s\text{-cluster})$$

$$= \frac{P(\text{site belongs to } s\text{-cluster})}{P(\text{site belongs to any cluster})}$$

$$= \frac{n_s s}{\sum_s n_s s}$$

Then, defining S = average cluster size,

$$S = \sum_s \omega_s s = \sum_s \left(\frac{n_s s^2}{\sum_s n_s s} \right)$$

$$\sum_s n_s s^2 = (1-p)^2 \sum_s s^2 p^s$$

$$= (1-p)^2 \left(p \frac{d}{dp}\right)^2 \sum_s p^s$$

that $\left(p\frac{d}{dp}\right)^2 = p\frac{d}{dp} + p^2 \frac{d^2}{dp^2}$ and using the previous trick to calculate the sums, we obtain

$$S = \frac{1+p}{1-p}$$

and note that S diverges at $p = p_c = 1$.