Question 1

Solution:→

$$F(M) = \frac{1}{2} r M^2 + u M^4 - h M$$

$M \in [-\infty, \infty]$ u weakly dependent on T

$r = a(T - T_c)$ $h \rightarrow$ magnetic field.

case (a) (i) $T > T_c \Rightarrow r > 0$

if r is positive the quartic term can be ignored and the minimum of $F(M) \Rightarrow$

$$\frac{dF(M)}{dM} = rM - h = 0$$

$$M \Rightarrow \frac{h}{r}$$

$M \Rightarrow \frac{h}{r} \Rightarrow F(M) \rightarrow$ Minimum.

so here $h \rightarrow 0$ so $M \rightarrow 0$ {Magnetization Goes to zero}

(ii) if $T < T_c$ $r < 0$

a quartic (term)(power 4) with a positive value of u is required to keep Magnetization finite.

so Now.

$$\frac{dF(M)}{dM} = rM + 4uM^3 - h = 0$$

$$4uM^3 + rM - h = 0$$

if $h = 0$

$$(4uM^2 + r)M = 0 \quad \text{so}$$

either $M = 0$

or

$$M^2 = -\frac{r}{4u}, \quad M = \sqrt{\frac{-r}{4u}}$$

so finally

$$M = \begin{cases} 0 & \text{if } T > T_c \\ \sqrt{\frac{-r}{4u}} & T < T_c \end{cases}$$

(b) <u>sol</u> given for $h = 0$.

$$C \sim |T - T_c|^{-\alpha} \qquad \text{as } T \to T_c \quad \text{what } \alpha = ?$$

$$c \propto \frac{1}{|T-T_c|^{-\alpha}}$$

$$F(M) = \begin{cases} 0 & \text{if } T > T_c \\ -\frac{r^2}{16u} & \text{if } T < T_c \end{cases}$$

$$F(M)\Big|_{M=\sqrt{\frac{-r}{4u}}} = \frac{1}{2} r \left(\frac{-r}{4u}\right) + u\left(\frac{r^2}{16u^2}\right).$$

$$= \frac{-r^2}{8u} + \frac{r^2}{16u}$$

$$= -\frac{r^2}{16u}$$

$$E = -\frac{\partial \ln Z}{\partial \beta} \;; \quad \frac{\partial}{\partial \beta} = -k_B T^2 \frac{\partial}{\partial T} \approx -k_B T_c \frac{\partial}{\partial r}$$

$$c = \frac{\partial E}{\partial T} = \begin{cases} 0 & \text{if } T > T_c \\ \frac{k_B}{8u} & T < T_c \end{cases}$$

So $C_{specific} = \begin{cases} 0 & T > T_c \\ \frac{k_B}{8u} & \text{if } T < T_c \end{cases}$

so its free of T

$$\boxed{\text{So} \quad \alpha = 0}$$

c) Sol At $T = T_c$ $M \sim h^{\delta}$

if $T = T_c$; $r = 0$

So $\frac{dF(M)}{dM} = 4uM^3 - h = 0$

$= M^3 = \frac{h}{4u}$

$M = \left(\frac{h}{4u}\right)^{1/3} \simeq h^{\delta}$

$$\boxed{\text{So} \quad \delta = \frac{1}{3}}$$ Sol.