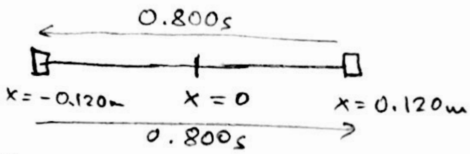
Name: 1B Discussion - Week 1

1. **Introducing Simple Harmonic Motion** *(YF 13th ed. 14.2)*. If an object on a horizontal, frictionless surface is attached to a spring, displaced, and then released, it will oscillate. If it is displaced 0.120m from its equilibrium position and released with zero initial speed, then after 0.800s its displacement is found to be 0.120m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude; (b) theperiod; (c) the frequency.

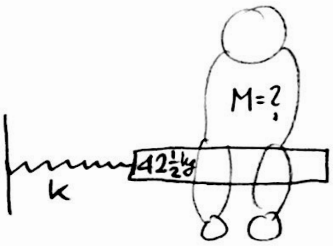


a) $A = ? = 0.120\,m$

b) $T = 2 \times 0.8 = 1.60\,s$

c) $f = \frac{1}{T} = \frac{5}{8}\,Hz$

2. **Weighing Astronauts** *(YF 13th ed. 14.15)*. This procedure has actually been used to "weigh" astronauts in space. A 42.5-kg chair is attached to a spring and allowed to oscillate. When it is empty, the chair takes 1.30s to make one complete vibration. But with an astronaut sitting in it, with her feet off the floor, the chair takes 2.54s for one cycle. What is the mass of the astronaut?



$$T = \frac{2\pi}{\omega} \quad ; \quad \omega = \sqrt{\frac{k}{m}}$$

$$T_1 = 2\pi\sqrt{\frac{m_1}{k}} = 1.30\,s$$

$$T_2 = 2\pi\sqrt{\frac{m_1 + M}{k}} = 2.54\,s$$

$$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \frac{m_1}{m_1 + M} = \left(\frac{1.3}{2.54}\right)^2$$

$$\Rightarrow m_1 \left(\frac{2.54}{1.3}\right)^2 - m_1 = M$$

$$\Rightarrow M = 120\,kg$$

3. **March Madness** *(YF 13th ed. 14.28)*. A cheerleader waves her pom-pom in SHM with an amplitude of 18.0 cm and a frequency of 0.850Hz. Find (a) the maximum magnitude of the acceleration and of the velocity; (b) the acceleration and speed when the pom-pom's coordinate is 9 cm (c) the time required to move from the equilibrium position directly to a point 12.0 cm away. (d) Which of the quantities asked for in parts (a), (b), and (c) can be found using the energy approach used in Section 14.3, and which cannot? Explain.

a) $A_0 = 18.0\,cm \quad , \quad f = 0.850\ Hz = \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f$

v is max. @ $x = 0 \quad \Rightarrow \frac{1}{2}mv_m^2 + \cancel{\frac{1}{2}kx^2} = \frac{1}{2}kA_0^2 \Rightarrow v_m = \omega A = 0.961\,\frac{m}{s}$

a is max. @ $x = A_0 \quad \Rightarrow -kx = F_{net} = ma \Rightarrow a_m = \omega^2 A = 5.13\,\frac{m}{s^2}$

b) $x_1 = 9.00\,cm$

$a(x_1) = ? = -\omega^2 x = -2.57\,m/s^2$

$v = \pm\,\omega\sqrt{A^2 - x^2} = \pm\,0.832\,m/s$

c) $x = A\cos(\omega t + \phi) \quad , \quad x = 0 \text{ @ } t = 0 \,,\ \phi = -\frac{\pi}{2}$

$x = A\sin(\omega t) \Rightarrow t = \sin^{-1}(x/A)/\omega = 0.137\,s$

d) Energy:

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$\Rightarrow$ find v, x ; not t

fix x to find a ; direction of a ambiguous

Force:

$ma = -kx$

$\Rightarrow$ find a, x ; integrate acc. to find v.

4. **Introducing Damping** *(YF 13th ed. 14.59)*. An unhappy 0.300-kg rodent, moving on the end of a spring with force constant ($k = 2.5\,N/m$) is acted on by a damping force $F_x = -bv_x$.
(a) If the constant b has the value $0.9\,kg/s$ what is the frequency of oscillation of the rodent?
(b) For what value of the constant b will the motion be critically damped?

a) $\omega' = \sqrt{(k/m) - (b^2/4m^2)} = 2.47\ rad/s \quad ; \quad b = 0.9\ kg/s$
$k = 2.5\ N/m$

$f' = \frac{\omega'}{2\pi} = 0.393\ Hz$

b) @ critical damping $b = 2\sqrt{km} \quad ; \quad k = 2.5\ N/m$

$\Rightarrow b_c = 1.73\ kg/s$

$\Rightarrow b < b_c \Rightarrow$ the poor rodent will undergo underdamped oscillation

$A(t) \propto e^{-\frac{bt}{2m}} \quad , \quad x(t) = A(t) = x e^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$