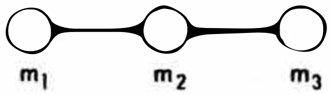
Name:

Practice Questions for the Comprehensive Exam - Day 1

18. Three rigid spheres are connected by light, flexible rods with relative masses as shown below:



$m_1 : m_2 : m_3 = 1:2:1$

Describe all the normal modes of the system and state whatever you can about the relative frequencies.

40. A positron (energy E_+, momentum $\mathbf{p}_+$) and an electron (energy E_-, momentum $\mathbf{p}_-$) are produced in a pair-creation process.

(a) What is the velocity of the frame in which the pair has zero momentum (barycentric frame)?

(b) Deduce the energy either particle has in this frame, and

(c) give an expression for the magnitude of the relative velocity between the particles, i.e. the velocity of one particle as seen by an observer attached to the other.

Consider two flavours of massive neutrinos, denote $|\nu_e\rangle$ the electron neutrino flavour eigenstate and $|\nu_\mu\rangle$ the muon neutrino flavour eigenstate. These are related to the energy eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ by

$$\begin{aligned} |\nu_e\rangle &= \cos(\theta)\,|\nu_1\rangle - \sin(\theta)\,|\nu_2\rangle \\ |\nu_\mu\rangle &= \sin(\theta)\,|\nu_1\rangle + \cos(\theta)\,|\nu_2\rangle \end{aligned}$$

a) Show that flavor eigenstates and energy eigenstates are related by a unitary transformation.

b) The energy of the eigenstate $|\nu_i\rangle$ is

$$E_i = \sqrt{\vec{p}^2 c^2 + m_i^2 c^4}, \quad i = 1, 2$$

Assume that an electron neutrino is produced in the sun with momentum $\vec{p}$ such that $|\vec{p}| >> m_i c$. Find the probability for the electron neutrino to oscillate into a muon neutrino after travelling a distance L.

18. A quantum-mechanical system in the absence of perturbations can exist in either of two states 1 or 2 with energies E_1 or E_2. Suppose that it is acted upon by a time-independent perturbation

$$V = \begin{pmatrix} 0 & V_{12} \\ V_{21} & 0 \end{pmatrix},$$

where $V_{21} = V_{12}^*$. If at time $t = 0$, the system is in state 1, determine the amplitudes for finding the system in either state at any later time.

35. Consider the scattering of a particle by a simple cubic structure with lattice spacing d. The interaction with the lattice points is

$$V = \frac{-2\pi a\hbar^2}{m} \sum_i \delta(\mathbf{r} - \mathbf{r}_i).$$

Treat the scattering in Born approximation. Show from your result that the condition for nonvanishing scattering is that the Bragg law be satisfied.

Problem 3.3. Consider a particle of charge e and mass m in constant, crossed $\mathbf{E}$ and $\mathbf{B}$ fields:

$$\mathbf{E} = (0, 0, E), \quad \mathbf{B} = (0, B, 0), \quad \mathbf{r} = (x, y, z). \tag{3.2}$$

a) Write the Schrödinger equation (in a convenient gauge).

b) Separate variables and reduce it to a one-dimensional problem.

c) Calculate the expectation value of the velocity in the x-direction in any energy eigenstate (sometimes called the drift velocity).

Problem 3.4. A particle of mass m and charge q sits in a harmonic oscillator potential $V = k(x^2 + y^2 + z^2)/2$. At time $t = -\infty$ the oscillator is in its ground state. It is then perturbed by a spatially uniform time-dependent electric field

$$\mathbf{E}(t) = Ae^{-(t/\tau)^2}\hat{\mathbf{z}} \tag{3.3}$$

where A and τ are constants. Calculate in lowest-order perturbation theory the probability that the oscillator is in an excited state at $t = \infty$.