Physics 1A: Physics for Scientists and Engineers: Mechanics **Winter 2015**

Discussion Week 2: Week 2

Exercise 1: (a) If the displacement of a particle as a function of time is given by $x(t) = \alpha t^3 - \frac{\beta}{t}$, where α and β are constants, find the velocity and acceleration of the particle as functions of time.

(b) If the acceleration of a particle is given by $a(t) = A\sin(\omega t)$, and the particle is initially at rest at the origin, find the velocity and position of the particle as a function of time.

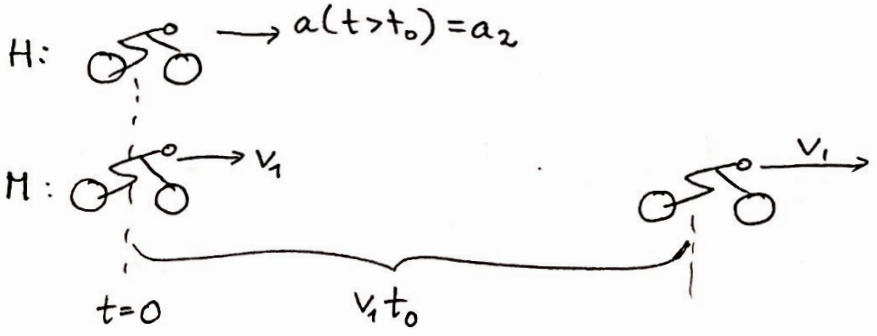
a) $x(t) = \alpha t^3 - \frac{\beta}{t}$, $v = \frac{\partial x}{\partial t} = 3\alpha t^2 + \frac{\beta}{t^2}$, $a = \frac{dv}{dt} = 6\alpha t - \frac{2\beta}{t^3}$

b) $a(t) = A\sin(\omega t)$

$$v(t) = \int_0^t A\sin(\omega t)\,dt = \frac{-A}{\omega}\cos(\omega t) + \frac{A}{\omega}$$

$$x(t) = \int_0^t \left(-\frac{A}{\omega}\cos(\omega t) + \frac{A}{\omega}\right)dt = \frac{-A}{\omega^2}\sin(\omega t) + \frac{A}{\omega}t$$

Exercise 2: Hayden and Matthew are riding around the neighborhood on their scooters. Hayden is at rest when Matthew passes him moving at a constant speed of v_1. After time t_0, Hayden decides to chase after Matthew, accelerating at a_2. How much time must Hayden accelerate before he is side-by-side with Matthew?



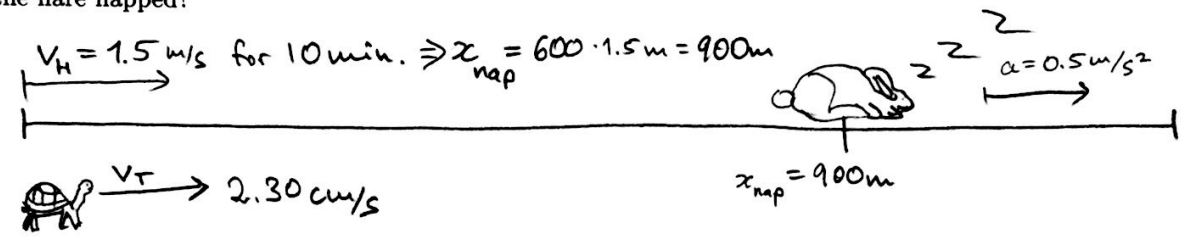
$x_H(t) = x_M(t)$ for side-by-side. Find t.

$$\left.\begin{aligned} x_H(t) &= x_0 + v_0 t + \tfrac{1}{2}at^2 \\ x_M(t) &= v_1 t_0 + v_1 t \end{aligned}\right\} t > t_0$$

$$\Rightarrow x_H = x_M = \tfrac{1}{2}a_2 t^2 = v_1 t_0 + v_1 t \Rightarrow \tfrac{1}{2}a_2 t^2 - v_1 t - v_1 t_0 = 0$$

$$\Rightarrow \boxed{t_{1,2} = \frac{v_1 \pm \sqrt{v_1^2 + 2a_2 v_1 t_0}}{a_2}}$$

Exercise 3: A tortoise and a hare are having a 1000-meter race. The tortoise runs the race at a constant speed of 2.30 cm/s. The hare moves at an average speed of 1.50 m/s for 10.0 minutes and then decides to take a nap. After waking up from the nap, the hare recognizes that the tortoise is about to cross the finish line and immediately accelerates from rest with a constant acceleration of 0.500 m/s^2 for the remaining distance of the race. If the tortoise wins by a hair (no pun intended), then what is the time in hours that the hare napped?



$$T_{total} = \frac{L}{v_T} = t_{jog} + t_{nap} + t_{sprint}$$

$t_{jog} = 10\,\text{min} = 600\text{s}$

$t_{sprint} = ?$, $\Delta x = 100\text{m} = \cancel{v_0 t}^{\,0} + \frac{1}{2} a t_s^2$, so $t_s = \sqrt{400} = 20\text{s}$

$$t_{tortoise} = \frac{L}{v_T} = \frac{1000\text{m}}{0.023\,\text{m/s}} = 43478\text{s} = T_{total}$$

$$\Rightarrow t_{nap} = T_{total} - t_{jog} - t_{sprint} = 43.5\,\text{ks} - 600\text{s} - 20\text{s} =$$

$$= 42.9\,\text{ks} \approx 11.9\,\text{h}$$

$\Rightarrow$ hare is well-rested.