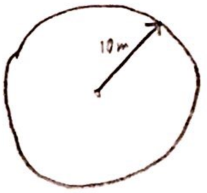
Physics 1A: Physics for Scientists and Engineers: Mechanics **Winter 2015**

Discussion 2: Week 3

Exercise 1: Hypergravity At its Ames Research Center, NASA uses its large 20-G centrifuge to test the effects of very large accelerations (hypergravity) on test pilots and astronauts. In this device, an arm of length $L = 10$ m rotates about one end in a horizontal plane, and the astronaut is strapped in at the other end. Suppose that he is aligned along the arm with his head at the outermost end. The maximum sustained acceleration to which humans are subjected in this machine is $a = 12.5g$. (a) How fast must the astronauts head be moving to experience this maximum acceleration? (b) What is the *difference* between the acceleration of his head and feet if the astronaut is 2.00 m tall? (c) How fast in rpm (rev/min) is the arm turning to produce the maximum sustained acceleration?



a) $\frac{v^2}{r} = a(r) \Rightarrow v = \sqrt{r \cdot a} = \sqrt{10 \cdot 12.5g} = 35 \text{ m/s}$

b) $\omega = \frac{d\theta}{dt} = \text{const.} = \text{angular frequency} = \left[\frac{\text{radians}}{\text{s}}\right]$

$v(r) = \omega r \Rightarrow a(r) = \omega^2 r \Rightarrow \omega^2 = \frac{a}{r} = \text{const.}$

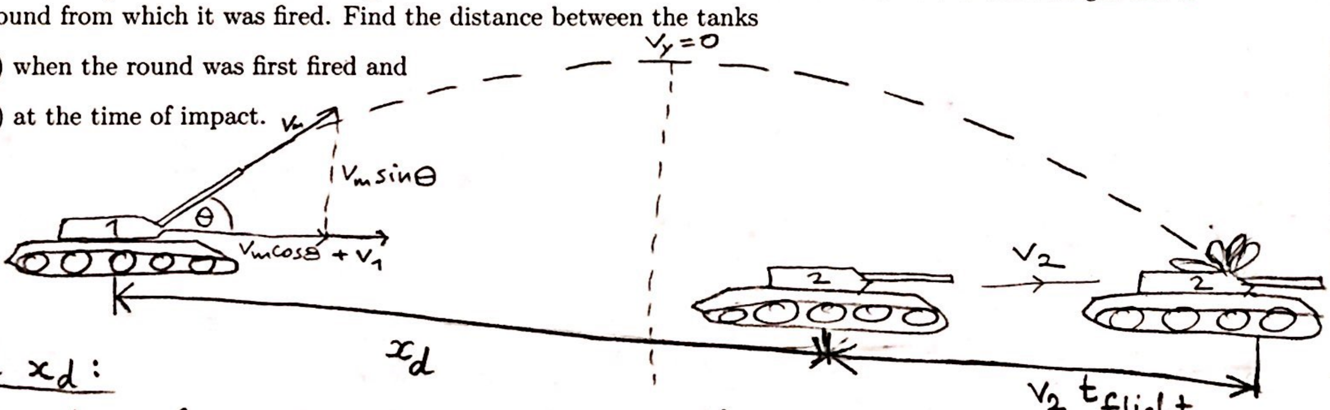
$\Rightarrow \frac{a(r_1)}{r_1} = \frac{a(r_2)}{r_2} \Rightarrow \frac{12.5g}{10\text{m}} = \frac{xg}{8\text{m}} \Rightarrow x = 10$ $\quad \Delta a = 2.5g$

c) $2\pi r \cdot N/60\text{s} = v = 35 \text{ m/s}$; taking $\pi = 3$, $N = 35$.

Exercise 2: Two tanks are engaged in a training exercise on level ground. The first tank fires a paint-filled training round with a muzzle speed of v_m at angle θ above the horizontal while advancing toward the second tank with a speed of v_1 relative to the ground. The second tank is retreating at v_2 relative to the ground, but is hit by the shell. You can ignore air resistance and assume the shell hits at the same height above ground from which it was fired. Find the distance between the tanks

(a) when the round was first fired and

(b) at the time of impact.



a) Find x_d:

$x_d + v_2 t_f = (v_m \cos\theta + v_1)\, t_f$

$\Rightarrow$ Need t_f:

$v_y(t_f/2) = 0 = v_m \sin\theta - g t_f/2$

$\Rightarrow t_f = \frac{2 v_m \sin\theta}{g}$

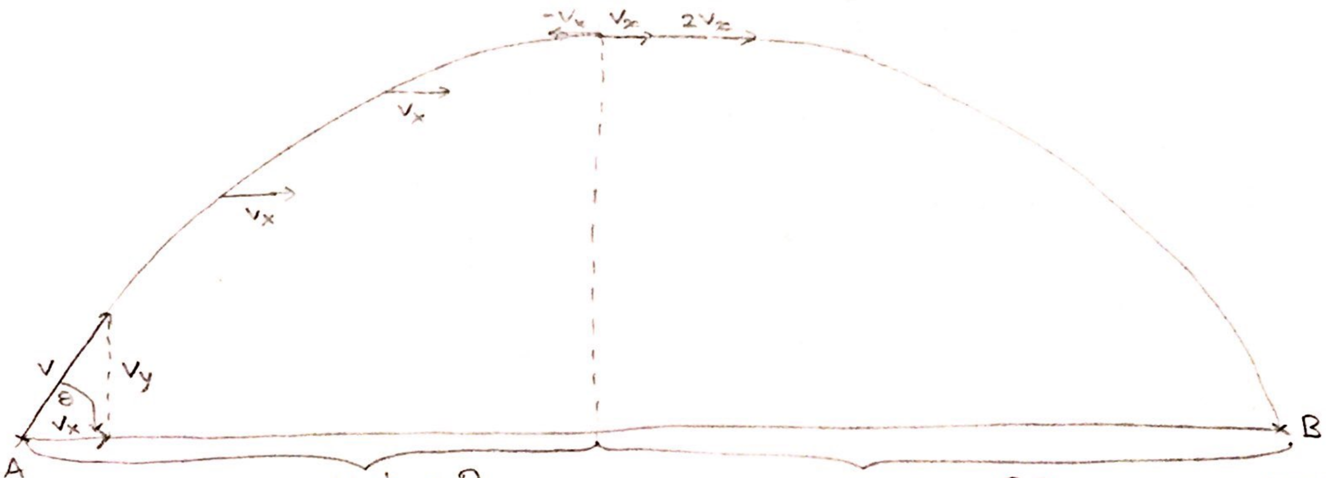
$\Rightarrow x_d = (v_m \cos\theta + v_1 - v_2)\, \frac{2 v_m \sin\theta}{g}$

b) Find $x_d + (v_2 - v_1)\, t_f =$

$= v_m \cos\theta \, t_f =$

$= \frac{2 v_m^2 \cos\theta \sin\theta}{g}$

Exercise 3: A projectile is fired from point A at an angle above the horizontal. At its highest point, after having traveled a horizontal distance D from its launch point, it suddenly explodes into two identical fragments that travel horizontally with equal but opposite velocities as measured *relative to the projectile just before it exploded*. If one fragment lands back at point A, how far from A (in terms of D) does the other fragment land?



$V_x t_0 = D$ $\qquad$ $3v_x t_0 = 3D$ $\qquad$ $\Rightarrow |\overline{AB}| = 4D$

Challenging Problem: A projectile is thrown from a point P. It moves in such a way that its distance from P is always increasing. Find the maximum angle above the horizontal with which the projectile could have been thrown. You can ignore air resistance.

If the total distance is always increasing, then the point of impact is the furthest point of the trajectory from the point of launch.

$$x(t) = \cancel{x_0}^{\,0} + v_x t + \cancel{\tfrac{1}{2} a t^2}^{\,0}$$

$$\Rightarrow x(t_{flight}) = V_x t_{flight}$$

$\Rightarrow$ find $x(\theta)$ and maximum point of $x(\theta)$.

$$t_{flight}(\theta) = ? \quad ; \quad v_y\left(\frac{t_f}{2}\right) = 0 = V_{oy} - g\, t_f/2 \Rightarrow t_f = \frac{2V_{oy}}{g} = \frac{2v\sin\theta}{g}$$

$$\Rightarrow x(t_f) = v\cos\theta\, t_f = \frac{v\cos\theta \cdot 2v\sin\theta}{g}$$

$$\Rightarrow \frac{\partial x(\theta)}{d\theta} = 0 \Rightarrow \cos^2\theta = \sin^2\theta$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$